Formal Uplift: Task Substitution

Tom Cunningham Parker Whitfill

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We describe a simple model of time allocation across tasks, with a few observations:

1. An inequality between three types of uplift: uplift on old tasks, uplift in value, and uplift on new tasks.
2. A graphical representation of the three types of uplift, illustrated against a demand curve.
3. A discussion of "Cadillac tasks", with examples, where uplift on new tasks greatly exceeds uplift in value.
4. A discussion of Hulten's theorem.

1 Setup

Consider a decision-maker who chooses how much of n tasks to complete, $\mathbf{x} = (x_1, \dots, x_n)$, to maximize an objective function $u(\mathbf{x})$ subject to a time budget constraint. Each task j requires t_j units of time per completion, and the decision-maker has total time T. The problem is

$$\max_{\mathbf{x} \geq 0} \; u(\mathbf{x}) \quad \text{subject to} \quad \sum_{j=1}^{n} t_j \, x_j \leq T \tag{1}$$

The objective u could represent utility, profit, or any other quantity the decision-maker seeks to maximize. We assume u is continuous, strictly quasi-concave, and strictly increasing, and that $t_j > 0$ for all j, so the optimum exists, is unique, and exhausts the time budget. Let $\mathbf{x}^*(\mathbf{t}, T)$ denote the optimal task allocation.

2 Old-Task and New-Task Uplift

Suppose AI decreases the time costs from $\mathbf{t}^0$ to $\mathbf{t}^1$, where $t_j^1 \leq t_j^0$ for all j. Let $\mathbf{x}^0 = \mathbf{x}^*(\mathbf{t}^0, T)$ and $\mathbf{x}^1 = \mathbf{x}^*(\mathbf{t}^1, T)$ denote the pre- and post-AI optimal allocations. We define two observable measures of uplift:

Uplift on old tasks. How many times faster could the pre-AI tasks be completed with AI?

$$\text{uplift on old tasks} = \frac{\sum_j t_j^0 \, x_j^0}{\sum_j t_j^1 \, x_j^0} \tag{2}$$

Uplift on new tasks. How many times longer would the post-AI tasks take without AI?

$$\text{uplift on new tasks} = \frac{\sum_j t_j^0 \, x_j^1}{\sum_j t_j^1 \, x_j^1} \tag{3}$$

3 What Does Uplift in Value Mean?

In the associated note, we claim

$$\text{uplift on old tasks} \leq \text{uplift in value} \leq \text{uplift on new tasks}. \tag{4}$$

But what does "uplift in value" mean? In economics, there are a few relevant possible definitions. We will show that, under some conditions, the above holds for all of these notions.

3.1 Utility-Based Uplift in Value

Let $u_0 = u(\mathbf{x}^0)$ and $u_1 = u(\mathbf{x}^1)$ denote the pre- and post-AI objective values. If u is cardinal – i.e., the level of u has intrinsic meaning, such as profits, number of papers produced, or revenue – and $u_0 > 0$, then we can define uplift in value directly as the ratio:

$$\text{uplift in value} = \frac{u_1}{u_0} = \frac{u(\mathbf{x}^1)}{u(\mathbf{x}^0)}. \tag{5}$$

This is plausibly the most intuitive notion of uplift, though it doesn't work if u is just a utility function that represents a person's preferences. In that case, u is only ordinal and we will have to refer to the following alternative definition.

3.2 Expenditure-Based Uplift in Value

Define the expenditure function as the minimum time required to achieve objective level $\bar{u}$:

$$e(\mathbf{t}, \bar{u}) = \min_{\mathbf{x} \geq 0} \left\{ \sum_{j=1}^{n} t_j \, x_j \;\middle|\; u(\mathbf{x}) \geq \bar{u} \right\}. \tag{6}$$

The expenditure-based uplift in value is the ratio $e(\mathbf{t}^0, \bar{u})/e(\mathbf{t}^1, \bar{u})$: how much more time would it cost at old time costs to achieve objective level $\bar{u}$ compared to new time costs? This is well-defined for any ordinal u, but it is reference-dependent – it depends on the choice of $\bar{u}$. The two natural reference points are:

$$\frac{e(\mathbf{t}^0, u_0)}{e(\mathbf{t}^1, u_0)} \quad \text{(evaluated at the pre-AI objective level)} \tag{7}$$

$$\frac{e(\mathbf{t}^0, u_1)}{e(\mathbf{t}^1, u_1)} \quad \text{(evaluated at the post-AI objective level)} \tag{8}$$

3.3 Uplift Inequalities

Suppose u is homogeneous of degree one, i.e., $u(\alpha \mathbf{x}) = \alpha \, u(\mathbf{x})$ for all $\alpha > 0$.

Proposition 1. *If u is homogeneous of degree one, then all three definitions of uplift in value coincide:*

$$\frac{e(\mathbf{t}^0, u_0)}{e(\mathbf{t}^1, u_0)} = \frac{e(\mathbf{t}^0, u_1)}{e(\mathbf{t}^1, u_1)} = \frac{u_1}{u_0}. \tag{9}$$

A proof is given in Appendix A.

Corollary 1. *If u is homogeneous of degree one, then*

$$\textit{uplift on old tasks} \leq \underbrace{\frac{e(\mathbf{t}^0, u_0)}{e(\mathbf{t}^1, u_0)} = \frac{u_1}{u_0} = \frac{e(\mathbf{t}^0, u_1)}{e(\mathbf{t}^1, u_1)}}_{\textit{uplift in value}} \leq \textit{uplift on new tasks}. \tag{10}$$

A proof is given in Appendix A.

4 Graphical Intuition: Quasi-Linear Case

4.1 Setup

For graphical intuition, suppose u is quasi-linear in a numeraire good x_0:

$$u(\mathbf{x}) = x_0 + f(x_1, \dots, x_n),$$

where f is twice continuously differentiable, strictly concave, and strictly increasing. One natural interpretation of x_0 is leisure: every unit of time not spent on tasks $1, \dots, n$ contributes one-for-one to utility (i.e., the time cost of leisure is normalized to 1, both pre- and post-AI). The decision-maker's problem is

$$\max_{\mathbf{x} \geq 0} \; x_0 + f(x_1, \dots, x_n) \quad \text{subject to} \quad x_0 + \sum_{j=1}^{n} t_j \, x_j \leq T.$$

We assume T is large enough that $x_0 > 0$ at the optimum for every time cost vector $\mathbf{t}$ on the coordinate path connecting $\mathbf{t}^0$ and $\mathbf{t}^1$.

4.2 Uplift Inequality Restated

In Section 3, we assumed u homogeneous of degree one and stated the uplift sandwich as a ratio inequality. For graphical intuition, we now slightly modify assumptions: assuming quasi-linearity makes u cardinal, denominated in units of the x_0 good, and lets us restate the inequality in terms of differences rather than ratios. The analogous difference inequality is

$$\sum_{j=1}^{n} (t_j^0 - t_j^1)\, x_j^0 \;\leq\; u_1 - u_0 \;\leq\; \sum_{j=1}^{n} (t_j^0 - t_j^1)\, x_j^1. \tag{11}$$

Both bounds follow from revealed preference.[1]

4.3 Graphical Depiction of Inequality

To get some traction on this inequality, note we can equivalently write $u_1 - u_0$ as a sum of integrals over the demand curves:

Proposition 2.

$$u_1 - u_0 \;=\; \sum_{j=1}^{n} \int_{t_j^1}^{t_j^0} x_j\big(t_1^1, \dots, t_{j-1}^1, t_j, t_{j+1}^0, \dots, t_n^0\big)\; dt_j. \tag{12}$$

A proof is given in Appendix A.

Corollary 2. *Substituting Proposition 2 into the difference inequality,*

$$\sum_{j=1}^{n} (t_j^0 - t_j^1)\, x_j^0 \;\leq\; \sum_{j=1}^{n} \int_{t_j^1}^{t_j^0} x_j\big(t_1^1, \dots, t_{j-1}^1, t_j, t_{j+1}^0, \dots, t_n^0\big)\; dt_j \;\leq\; \sum_{j=1}^{n} (t_j^0 - t_j^1)\, x_j^1. \tag{13}$$

To build intuition for this object, we can focus on the case where a single t_j varies.[2]

$$(t_j^0 - t_j^1)\, x_j^0 \;\leq\; \int_{t_j^1}^{t_j^0} x_j(t_j)\, dt_j \;\leq\; (t_j^0 - t_j^1)\, x_j^1. \tag{14}$$

[1] For the lower bound: optimality of $\mathbf{x}^1$ at prices $\mathbf{t}^1$ gives $f(\mathbf{x}^1) - \sum t_j^1 x_j^1 \geq f(\mathbf{x}^0) - \sum t_j^1 x_j^0$, which rearranges to $u_1 - u_0 \geq \sum (t_j^0 - t_j^1) x_j^0$. The upper bound follows symmetrically from optimality of $\mathbf{x}^0$ at prices $\mathbf{t}^0$.

[2] The same geometric argument applies to each step of the coordinate path in the multidimensional case.

This is now very simple: all three terms are different ways of looking at the area under the demand curve for x_j between t_j^1 and t_j^0. The middle is the exact area; the outer two are rectangle approximations using the pre- and post-AI quantities, respectively. Figures 1 and 2 illustrate.

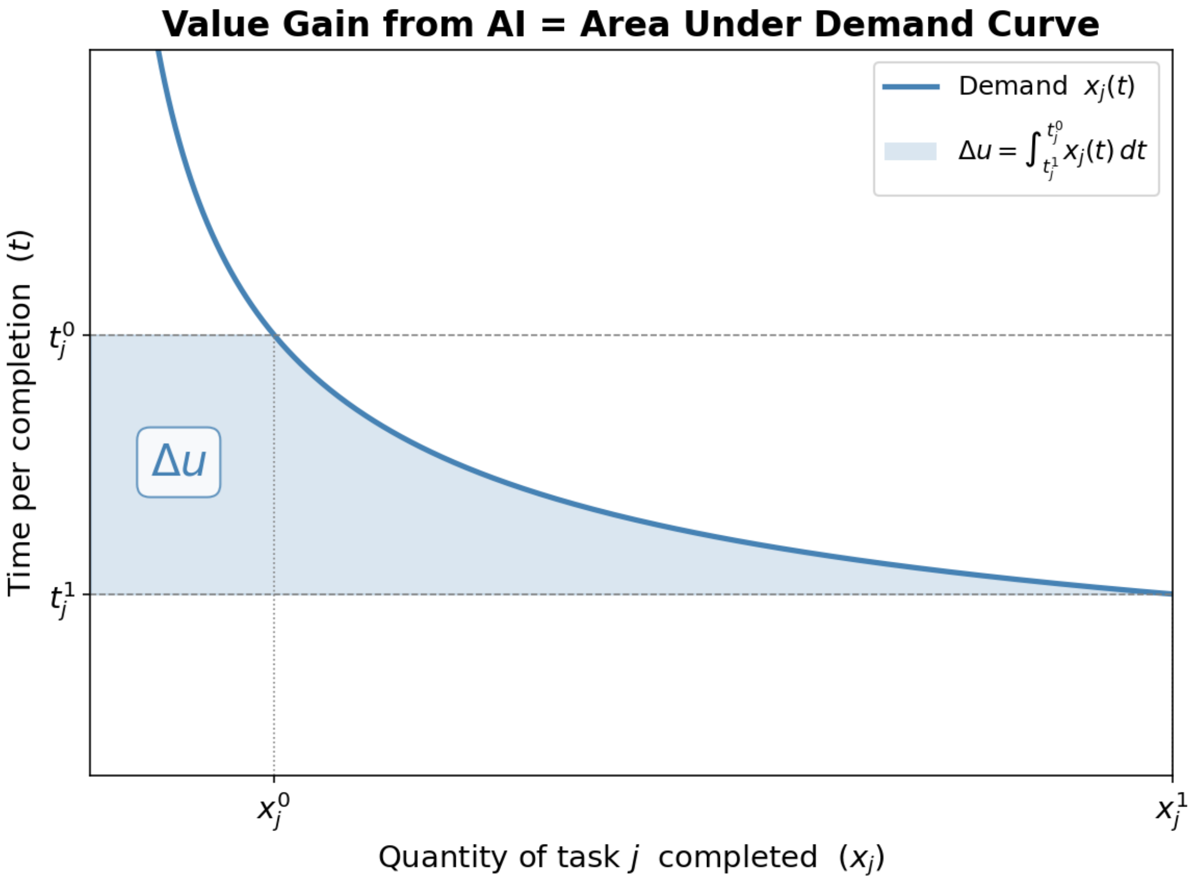

Figure 1: The value gain from reducing the cost of task j from t_j^0 to t_j^1 equals the area under the demand curve.

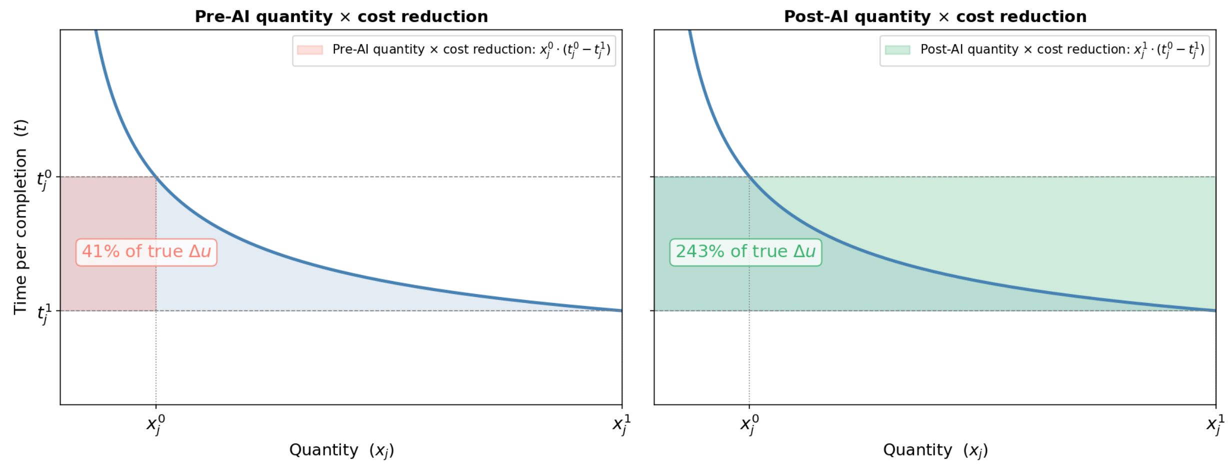

Figure 2: Rectangular approximations to the value gain. The left panel uses the pre-AI quantity x_j^0; the right panel uses the post-AI quantity x_j^1.

4.4 Cadillac Tasks

The worst case for the uplift on new tasks rectangle is "Cadillac tasks": tasks the decision-maker would never have done without AI because the time cost was prohibitive, but now does in large quantities because AI has made them cheap. In this case $x_j^0 \approx 0$ while x_j^1 is large, so the new-task rectangle can overstate the true value gain by an arbitrarily large factor.

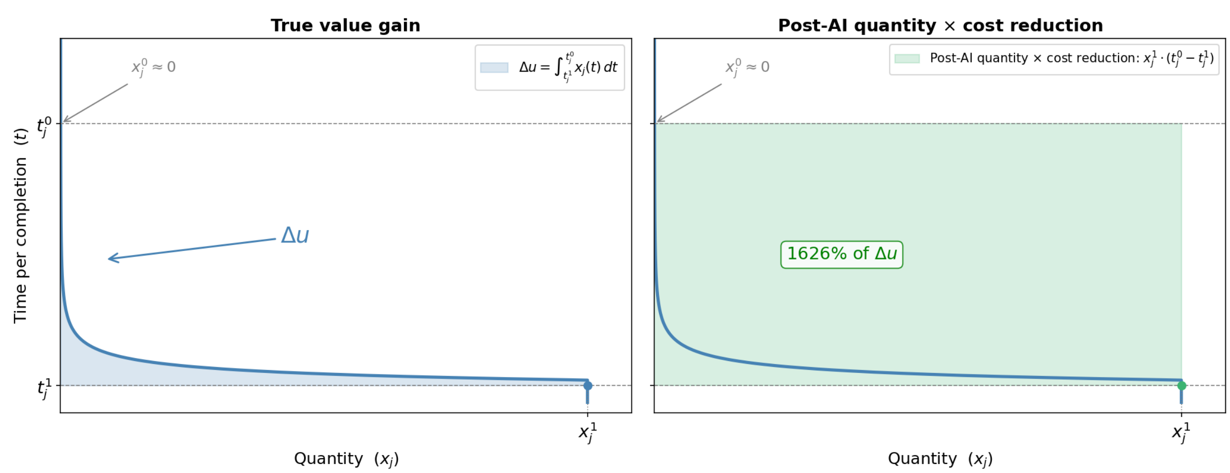

Figure 3: A Cadillac task: the demand curve is L-shaped, so $x_j^0 \approx 0$ while x_j^1 is large. The new-task rectangle can overstate $u_1 - u_0$ by an arbitrarily large factor.

We suspect Cadillac tasks are common in the AI setting. For example, AI coding agents can now complete software projects that would have taken an unskilled person months of effort. As a personal example, one of us began using Claude Code to build personal apps once Opus 4.5 could one-shot a working application. Earlier cost reductions (Cursor, Claude Opus 4) were not enough to justify the effort.

It would be a mistake to conclude that his productivity massively increased. It can *feel* like a huge increase: something that previously would have cost months now costs minutes. But in terms of value, the change is modest. The fact that he was doing essentially no personal software development before is itself evidence that the value of those tasks was low relative to their cost.

Cadillac tasks may help explain the disconnect between how productive people *feel* and what shows up in aggregate data. Workers report feeling 2–10× more productive with AI, yet GDP and TFP growth remains modest. Cadillac tasks offer a partial explanation: large time savings on tasks with low marginal value.

4.5 Hulten's Theorem

A common alternative approximation in the macro literature is Hulten's theorem, which weights log cost reductions by initial expenditures. For a change in costs across all tasks, it gives

$$u_1 - u_0 \;\approx\; \sum_{j=1}^{n} t_j^0\, x_j^0\, \ln\!\big(t_j^0/t_j^1\big)\,. \tag{15}$$

This is equivalent to approximating the area under the demand curve by using uplift on old tasks but instead of assuming the level is constant, we assume demand is unit elastic: if prices fall by 1%, demand rises by 1%. Figure 4 shows this graphically.

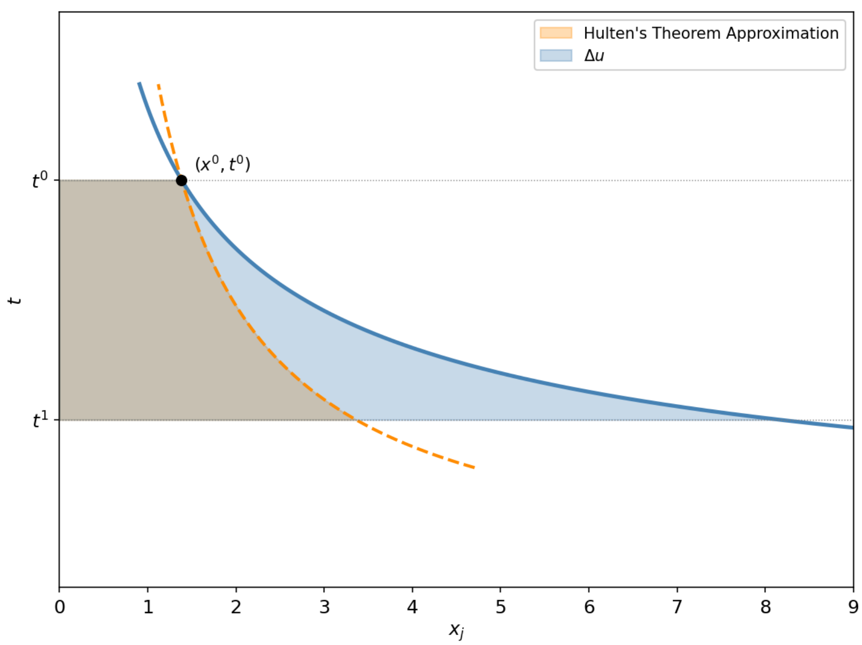

Figure 4: True value gain (blue, area under actual demand $x_j(t)$) versus Hulten's approximation (orange, area under the unit-elastic reference curve $\tilde{x}(t) = t^0 x^0/t$ that passes through the initial point (x^0, t^0)).

A Proofs

Lemma 1. *If u is homogeneous of degree one, then $e(\mathbf{t}, \bar{u}) = \bar{u} \cdot e(\mathbf{t}, 1)$ for all $\bar{u} > 0$.*

Proof. Let $\mathbf{x}^h$ solve $e(\mathbf{t}, 1) = \min\{\sum_j t_j x_j : u(\mathbf{x}) \geq 1\}$. By HD1, $\bar{u} \cdot \mathbf{x}^h$ achieves utility $\bar{u}$ at cost $\bar{u} \cdot e(\mathbf{t}, 1)$; conversely, if $\hat{\mathbf{x}}$ achieves $u(\hat{\mathbf{x}}) \geq \bar{u}$ at lower cost, then $\hat{\mathbf{x}}/\bar{u}$ achieves utility ≥ 1 at cost below $e(\mathbf{t}, 1)$, a contradiction. □

Proof of Proposition 1. Immediate from Lemma 1: $e(\mathbf{t}^0, \bar{u})/e(\mathbf{t}^1, \bar{u}) = e(\mathbf{t}^0, 1)/e(\mathbf{t}^1, 1)$ for any $\bar{u}$, and budget exhaustion at the optima gives $T = u_0 \cdot e(\mathbf{t}^0, 1) = u_1 \cdot e(\mathbf{t}^1, 1)$, so the common ratio equals u_1/u_0. □

Proof of Corollary 1. This is the classical Konüs sandwich theorem: under homothetic preferences (a fortiori under HD1), the Laspeyres and Paasche indices bracket the true (Konüs) index. See Konus [1939, p. 20] for the original statement and Diewert [2008, p. 201, eq. (46)] for a modern textbook treatment. □

Proof of Proposition 2. This is the standard envelope-theorem result for the change in indirect utility in the quasi-linear case; see Slesnick [2008, p. 153, eq. (1)]. □

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